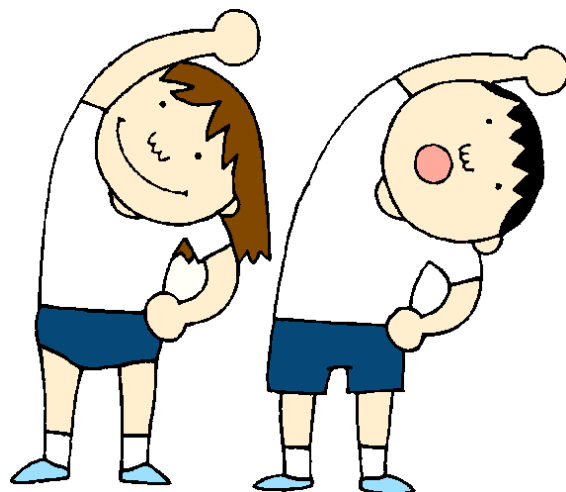


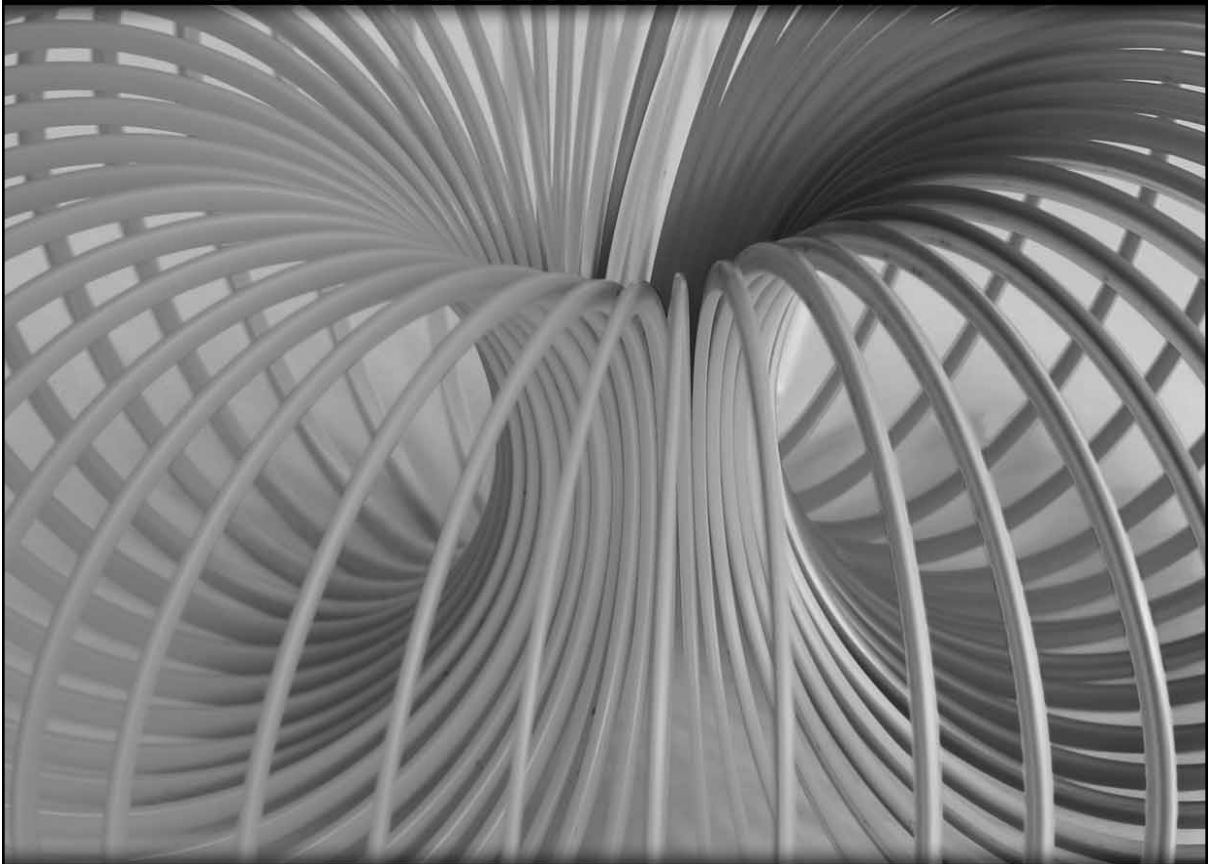
Mathematical Stretches: Math Warm-Ups to Begin the Day



Laney Sammons
NCTM Conference
San Diego 2010

Math *Stretches*

Building Conceptual Understanding



Author

Laney Sammons

 SHELL EDUCATION

Figure 1.1: Guided Math Menu of Instruction

Guided Math: Menu of Instruction
<p>Daily: Classroom Environment of Numeracy—A classroom should be a community where students are surrounded by mathematics. This includes real-life math tasks, data analysis, math word walls, instruments of measurement, mathematical communication, class-created math charts, graphic organizers, calendars, and evidence of problem solving.</p>
<p>Daily: Calendar Math and Morning Work—This daily appetizer prepares the palate for the “Your Choice” entrées below with calendar activities, problems of the day, data work, incredible equations, review of skills to be maintained, and previews of skills to come.</p>
<p>Your Choice: Whole-Class Instruction—The following are excellent teaching strategies to use when students are working at the same level of achievement: introducing lessons with a mini lesson or an activating strategy, teacher modeling and think-alouds, read-alouds of math-related literature, organizing a Math Huddle, reviewing previously mastered skills, setting the stage for Math Workshop, and using written assessments.</p>
<p>Your Choice: Small-Group Instruction—Students are instructed in small groups with changing composition based on their needs. The individualized preparation for these groups offers tantalizing opportunities to introduce new concepts, practice new skills, work with manipulatives, provide intensive and targeted instruction to struggling learners, introduce activities that will later become part of Math Workshop, conduct informal assessments, and reteach based on student needs.</p>
<p>Your Choice: Math Workshop—Students are provided with independent work to complete individually, in pairs, or in cooperative groups. The work may be follow-up from whole-class or small-group instruction, practice of previously mastered skills, investigations, math games, Math Journals, or interdisciplinary work.</p>
<p>Daily: Conferencing—To enhance learning, teachers confer individually with students, informally assess their understandings, provide opportunities for one-on-one mathematical communications, and determine teaching points for individual students, as well as for the class.</p>
<p>Daily: Assessment—Be sure to include a generous helping of assessment <i>for</i> learning to inform instruction, with a dollop of assessment <i>of</i> learning to top off each unit.</p>

How Did You Use Math Yesterday? *Stretch*

Using words or pictures, tell about one way you used math yesterday. Add your initials to your response.

Introduction *(cont.)*



Using Math Stretches to Promote Mathematical Literacy *(cont.)*

Planning Math Stretches *(cont.)*

The Math Stretches provided in this book offer examples of activities teachers can use to stimulate students' mathematical thinking as the day begins. Teachers are not limited to the activities included here. They may create engaging stretches specifically designed to fit the needs of their classes. When creating Math Stretches, it is important to keep in mind their characteristics:

- They are very brief.
- They can be completed by students independently.
- They prompt students to think mathematically.
- They encourage students to make mathematical connections to themselves, to other math concepts, and/or to math in other content areas.
- They generate mathematical communication.
- They provide opportunities to clarify understanding of mathematical vocabulary.
- They offer students repeated chances to revisit important mathematical big ideas.

Some teachers find it convenient to repeatedly use the same five stretches each week, assigning one regularly for each day of the week (Figure 2). Students learn the schedule and know what to expect as they enter the classroom. An example of a week's worth of morning Math Stretches for a third grade class is shown in Figure 2 below.

Figure 2: Weeklong Plan of Morning Math Stretches

Day of the Week	Morning Stretch	Topic
Monday	Number of the Day	180
Tuesday	What's Next?	1, 3, 9, 27, ...
Wednesday	How Did My Family Use Math Last Night?	Real-life mathematical connections
Thursday	A _____ Makes Me Think Of...	Multiplication
Friday	Data Collection	Where would you rather go on a field trip? <ul style="list-style-type: none">• science museum• planetarium• aquarium

Introduction *(cont.)*



Using Math Stretches to Promote Mathematical Literacy *(cont.)*

Math Huddles to Discuss the Math Stretch *(cont.)*

As students become more proficient at sharing their mathematical ideas related to the Math Stretch, teachers step back to become facilitators of the Math Huddle. Broad, open-ended questions posed by the teacher may start the discussion or move it along if it stalls. Other than that, the teacher ensures that the talk remains focused and that all students feel comfortable participating.

Eventually, when students have demonstrated their understanding of the process, they are asked to record their own analysis in their Math Journals *prior* to the class huddle. Writing ideas in their journals prior to the discussion helps students to formalize their ideas so that they can actually see, consider, and revise them if needed (O’Connell 2007, 134). This in no way eliminates the need for a discussion, however.






Students bring their journals to the meeting area as they share what they learned from the Math Stretch responses. As discussion about these responses prompts some students to change their ideas or expand them, they are encouraged to make these changes in their Math Journals. At this point, students become the primary participants in the Math Huddle, but the teacher does not abandon the discussion. The teacher must be present to help correct misconceptions and to continue the ongoing informal assessment of individuals and the class as a whole.

Figure 3 Types of Math Huddle Discussions

Discussion Type	Teacher Role	Level of Teacher Support
Think-Aloud	<ul style="list-style-type: none">• Describes the observations and inferences he or she has made• Thinks aloud to explain interpretations and mathematical reasoning• Models the use of relevant mathematical vocabulary• Records analysis on chart paper for future reference	High
Guided	<ul style="list-style-type: none">• Uses carefully crafted questions to guide discussion of observations and inferences from the Math Stretch• Occasionally rephrases student comments to model relevant mathematical vocabulary and concepts• Helps students consider the appropriateness of the numerical, graphic, or linguistic representations• Records mathematical ideas discussed on chart paper for future reference	Moderate to High
Facilitated	<ul style="list-style-type: none">• Poses open-ended questions to stimulate discussion by students• Ensures that discussion stays focused on observations, inferences, and choices of representation• Encourages recording of analysis on chart paper or in individual student Math Journals	Minimal to Moderate
Independent	<ul style="list-style-type: none">• Directs students to record observations, inferences, and choice of representation in Math Journals before discussion• Supports student discussion of journal reflections, if necessary, by focusing discussion or correcting misconceptions	Minimal

Number of the Day *Stretch*




Standards

-  Understands that numerals are symbols used to represent quantities or attributes of real-world objects
-  Understands symbolic, concrete, and pictorial representations of numbers
-  Understands basic whole number relationships
-  Adds and subtracts whole numbers
-  Counts whole numbers

Overview

With the Number of the Day Stretch, students are asked to represent a teacher-selected “number of the day” in multiple ways that may include, but are not limited to, composing a problem to represent the number, drawing objects to represent the number, and demonstrating an understanding of place value through the representation of the number. Teachers determine the number based on the grade-level curriculum and knowledge of their students.

Materials

-  chart paper
-  markers
-  assortment of manipulatives for counting

Warming Up for the Stretch

Introduce students to the Number of the Day activity during Calendar Board instruction. Teachers should model how to find alternate ways of expressing a “number of the day.” This is best done through a think-aloud. Since some students may need to work with manipulatives to complete this task independently, teachers should demonstrate how to use manipulatives (base ten blocks, linking cubes, or place value boards) to create models of the number that then can be expressed with words, numerals, symbols, or pictures.

In subsequent days, repeat the task during Calendar Board using other numbers of the day and with increased student involvement in suggesting ways to represent the number. Display the Number of the Day charts created by the class during these lessons in the classroom for student reference when the activity becomes a Math Stretch to be completed by students independently. Teachers should determine the number of the day for this task based on the grade-level curriculum and the needs of their students.







Number of the Day *Stretch* (cont.)

Stretch Procedures

1. Display the Number of the Day chart. For nonreaders, designate a reader or use an audio recording of the directions.
2. Have students record with words, numerals, symbols, or pictures a representation of the number of the day. Then students add their initials. Students should choose a way to represent the number that is different from their classmates' responses. Manipulatives may be used to help students discover alternative representations of the number.
3. Once all students have contributed to the Number of the Day chart, call the class together for a Math Huddle to discuss the representations of the number that are displayed on the chart. Use the questions below to aid in this discussion.

Suggested Questions for Informal Assessment: Math Huddle

**Level of
Teacher
Support**

-  Did you notice all the different ways we have represented the number of the day? Can you show us a representation using manipulatives? Is there a representation that you wonder about?
-  Why do you wonder about that representation? Do you think it accurately represents the number of the day? Why? Does anyone else question that representation?
-  Who would like to tell us about your mathematical thinking in coming up with your choice? Do others agree or disagree? Why?
-  How does understanding place value help us create representations of numbers? How does understanding addition and subtraction help us?
-  What connections do you make when you think of today's number? (*Answers might include, "That's my age," "That's how much money I have in my piggy bank," "That's how many students are in the class," or "That's how many fingers and toes I have."*)
-  Why do you think it is important to be able to represent numbers in different ways? When do we usually use number words to represent numbers? When do we use numerals? When do we use pictures or diagrams? When do we use number sentences? Why do we sometimes choose one method of representation rather than another?

Number of the Day *Stretch* (cont.)



What It Looks Like: Stretch Snapshot

The Number of the Day Stretch offers teachers an easy way to assess both their students' number sense as well as their ability to represent numbers in multiple ways. Assessing students solely on the representations they create for the Number of the Day may prove to be misleading. Especially with young children, errors in actual representation may occur even when conceptual understanding exists. Since young students are easily distracted, especially in completing independent work, it is essential to engage in conversation with those who have minor errors to determine the causes of the errors.

The kindergarten class in this Stretch Snapshot is focusing on numbers up to ten. The teacher has encouraged students to use 5 and 10 as benchmark numbers. These students are beginning to see that 3 is two less than 5, and that 7 is two more than 5 or three less than 10. In this Math Stretch, the Number of the Day was 8.

At the start of the Math Huddle, the teacher noticed that Michael had drawn seven pennies on the chart in his attempt at representation. However, during previous observations of Michael as he counted objects, the teacher noted that Michael understood one-to-one correspondence and could count well beyond 10. He even seemed to understand the concept of using benchmark numbers. His teacher wondered whether his error in this task was due to a lack of understanding or was, instead, a careless error.

Teacher: *Young mathematicians, I am amazed by your work! Look how many ways you found to represent the number 8! Take a few minutes to look at our chart. (The teacher gives students an opportunity to examine the many representations created by the class.) Some of you chose to express the number 8 by drawing eight objects. Who chose to show the number this way? Let's see...Meagan, can you tell us about your work?*

Meagan: *I drew a pizza with eight slices. I counted each one as I drew it, so I know I have eight. See... 1, 2, 3, 4, 5, 6, 7, 8. (She carefully points to each slice as she counts.)*

Teacher: *Class, can you count with Meagan to be sure that she has eight slices?*

The class counts with Meagan as she points to each slice. The teacher is watching Michael to see if he is participating and counting each time a slice is touched, or instead is just rote counting. He appears to be correctly counting one-by-one as Meagan points to each slice.)

Teacher: *Michael, do you agree with Meagan?*

Number of the Day *Stretch* (cont.)



What It Looks Like: Stretch Snapshot (cont.)

- Michael:** *Yes. We just counted them, so we know there are eight—just like my eight pennies.*
- Teacher:** *You drew pennies! That makes me think of a connection. We've been talking about making trades with coins, haven't we? If I wanted to make a fair trade with you by giving you a nickel, how many pennies would you have to give me?*
- Michael:** *Five of them.*
- Teacher:** *Okay. Will you draw a circle around five of your pennies to show how many you could trade for a nickel? (Michael counts five pennies and draws a circle around them. Then, he looks puzzled.)*
- Teacher:** *What's the matter, Michael? (The teacher provides wait time as Michael looks back at his work.)*
- Michael:** *This isn't right. Eight is three more than five, but when I circled the five pennies, there are only two left.*
- Teacher:** *How can you figure out what's wrong?*
- Michael:** *I can count again. 1, 2, 3, 4, 5, 6, 7. That's only seven! I need to draw another penny.*
- Teacher:** *How do you know?*
- Michael:** *First, I knew that 8 was three more than 5. That didn't work—there were only two left after I circled the 5. So, I counted all of them again. I needed one more.*

In this Snapshot, the dialogue with Michael confirmed the teacher's earlier assessment that Michael understood one-to-one correspondence and, furthermore, is capable of using benchmark numbers.



Number of the Day *Stretch* (cont.)

Sample Chart

How many ways can we represent this number?

Use words, numbers, or pictures to represent this number. Add your initials to your representation.

8

Hand-drawn representations of the number 8:

- Group of 8 circles (RJ)
- Group of 8 circles (V.A.)
- Two hands (L.L.)
- Base ten blocks: 1 ten rod and 8 one units (TR)
- Group of 8 circles (Q.M.)
- Equation: $8+0$ (Q.M.)
- Group of 8 circles (C.T.)
- Equation: $10-2$ (C.T.)
- Pizza divided into 8 slices (M.P.)
- Group of 8 circles (C.T.)
- Group of 8 circles (I.L.)
- Group of 8 circles (I.L.)
- Group of 8 vertical lines (P.S.)
- Group of 8 circles (B.R.)
- Group of 8 circles (D.T.)
- Equation: $2+2+2+2$ (O.N.)
- Group of 8 circles (P.M.)
- Equation: $1+1+1+1+1+1+1+1$ (R.M.)
- Group of 8 circles (L.)
- Group of 8 circles (T.G.)
- Equation: $4+4$ (V.A.)
- Group of 8 circles (C.T.)
- Group of 8 circles (M.J.)
- Stack of 8 blocks (S.C.)

Write a Story *Stretch*

Write a story using either words or pictures that is represented by this number sentence. Add your name to your story.

$$5 + 7 = 12$$

Table of Contents



Acknowledgements	4	Algebra and Algebraic Thinking Stretches (<i>cont.</i>)	
Introduction	5	What Comes in Pairs? Stretch	79
Promoting Mathematical Literacy for the 21st Century with Math Stretches	5	Sorting Numbers (Even/Odd) Stretch	84
Mathematical Literacy	5	Geometry Stretches	
Current Mathematical Instructional Methods	5	I Spy Shapes! Stretch.....	90
Research-Based Best Practices for Teaching Mathematics	6	Create a Polygon That Has ____ Stretch.....	95
Making Connections	6	Right Angles Around Us Stretch	100
Mathematical Communication.....	8	Is It a Square? Stretch.....	105
Vocabulary for Mathematics.....	9	Diagram Description Stretch	110
Repetition for Mathematical Learning...	10	Measurement Stretches	
Guided Math: A Flexible Framework for Mathematics Instruction	13	What Takes Less Than a Minute? Stretch	115
Using Math Stretches to Promote Mathematical Literacy.....	20	What Takes More Than an Hour? Stretch	120
Planning Math Stretches.....	20	What Is Shorter Than a Foot? Stretch.....	125
Preparing Math Stretches.....	22	How Long Is Your Pencil? Stretch	130
Teaching Procedures for Math Stretches	23	How Many Feet from Our Classroom Door to the Water Fountain? Stretch.....	135
Math Huddles to Discuss the Math Stretch	25	Data Analysis and Probability Stretches	
Types of Math Stretches.....	27	Tally Chart Stretch.....	140
How to Use This Product	31	“Real” Graph Stretch	145
Correlation to Mathematical Standards	36	Pictograph Stretch.....	150
Number and Operations Stretches		Symbolic Graph Stretch	155
Number of the Day Stretch	39	Line-Plot Graph Stretch.....	160
What’s My Neighbor? Stretch.....	44	Across the Discipline Stretches	
How Many More to 100? Stretch	49	How Did My Family Use Math Last Night? Stretch	165
About How Many? Stretch	54	A ____ Makes Me Think of... Stretch	170
Write a Story Stretch.....	59	Numbers in the News Stretch	175
Algebra and Algebraic Thinking Stretches		Know and Want to Know Stretch	180
Skip Counting Stretch.....	64	What Have I Learned About ____? Stretch	185
What’s Next? Stretch.....	69	References	190
Input/Output Stretch	74	Contents of Teacher Resource CD	192

Resources

Contact Information

Laney Sammons

LaneyAS@aol.com

Resources

Guided Math: A Framework for Mathematics Instruction

*Math Stretches: Building Conceptual Understanding
(Levels K–2)*

Guided Math Group

<http://tech.groups.yahoo.com/group/guidedmath/>

Shell Education

<http://www.shelleducation.com>